



# Cogeneration versus natural gas steam boiler: A techno-economic model



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## HIGHLIGHTS

- Both cogeneration and conventional systems may significantly reduce the costs.
- One heat exchanger is used for the two states of the cogeneration system.
- Using dynamic programming, we obtain analytical formulas for the expected costs.
- The cogeneration system's ROI is usually no longer than 7 years.
- High variances of steam demands lead to profitability of the conventional system.

## ARTICLE INFO

### Article history:

Received 26 March 2014

Received in revised form 10 June 2014

Accepted 13 June 2014

Available online 7 July 2014

### Keywords:

Energy production

Cogeneration

Natural gas

Optimization

## ABSTRACT

Energy usage can constitute a substantial operational expense for corporations. To reduce expenses, corporations may seek out decentralized solutions for generating electricity, based on sustainable energy or on conventional energy resources. The main goal of this research is to resolve an organization's dilemma regarding whether to adopt a cogeneration system or to replace a conventional diesel steam boiler with a boiler fueled by natural gas. We analytically calculate the total expected initial setup and operational costs under the two models, and determine which model is preferable. We numerically show that implementation of a cogeneration system may yield rapid return-on-investment and may lead to cost savings of more than 25%, as compared with the conventional system. However, low electricity tariffs or high operation costs lead to slower return-on-investment and the conventional model becomes significantly better for short-term processes. Furthermore, low uncertainty of steam demands leads to profitability of the cogeneration model. On the other hand, if the total expected demand of one type of product (electricity or steam) is significantly greater than that of the other, then the conventional model becomes preferable.

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## 1. Introduction

Energy usage in corporations—e.g., in manufacturing, heating, cooling, lighting, and technology—can constitute a substantial operational expense [1]. Consequently, firms seek out reliable energy-saving solutions that can reduce their expenditures. The use of natural gas (NG) can enable organizations to enhance their efficiency while reducing emissions, thereby contributing to a cleaner environment [2].

By 2040, required net electricity generation worldwide is expected to increase to 39.0 trillion kilowatt-hours, from 20.2 trillion kilowatt-hours in 2010 [3]. So far, most countries are addressing this issue by increasing energy production capacity through the

construction of centralized power plants on the one hand, and reducing demand by investing in facilities' efficiency on the other hand. Construction of new central power plants has many disadvantages, including high capital cost, low efficiency, environmental impact, and utilization of open spaces.

There are numerous alternative, decentralized/distributed solutions for generating electricity from sustainable energy sources (photovoltaic energy, wind power, hydroelectric power) or conventional energy resources [4]. Sustainable energy solutions are not always practical, as they can require large areas of land, and they may not be able to supply continuous energy in unfavorable weather conditions. However, decentralized fossil fuel solutions (such as NG) can provide high efficiency, reduce environmental impact (as compared with reliance on other fossil fuels), provide a continuous energy supply, and lower dependency on foreign oil.

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The use of NG to power conventional equipment such as steam boilers decreases financial expenditure and reduces emissions, in comparison with the use of coal- or oil-based energy. Efficiency can be enhanced even further if NG is utilized through cogeneration technology. Cogeneration is the simultaneous production of power and usable heat, based on using one type of energy source such as NG [5]. In most cogeneration applications, the energy types produced simultaneously are electric and heat energies, such as electricity and steam. Generally, cogeneration systems create heat by utilizing the waste heat energy produced during electricity generation. As a result, a single cogeneration system can produce a given amount of electrical and thermal energy using less fuel than would be needed to generate the same quantities of both types of energy with separate conventional technologies.

The main goal of this research is to enable an organization to identify which of the following approaches will provide it with the greatest reduction in energy expenditures: adopting an NG-powered cogeneration system (that produces both electrical power and steam) or replacing a conventional diesel steam boiler with an NG boiler while deriving all electrical power from the grid. We develop a model for each scenario (the “cogeneration model” and the “conventional model”, respectively), calculating the scenario's profitability according to initial setup cost in addition to the operational costs corresponding to electricity and steam demand in each time period. Electrical bills have a structure of different tariffs, which can vary significantly according to season, day of the week and time of day. Implementation of a cogeneration system provides the organization with two electricity sources (grid or cogeneration), giving the organization the flexibility to decide when to turn the cogeneration system on or off, in accordance with tariff rates. Thus, with the appropriate capacity planning and operational strategy, the use of a cogeneration system has the potential to reduce the organization's energy expenditures. The disadvantage of the cogeneration model is the high setup cost.

Our model resolves the organization's dilemma while taking into account all available information that might influence the outcome of the investment. We find that with optimal initial configuration of the system, as well as an optimal operational policy regarding implementation of the system, a cogeneration system can be more profitable than a conventional NG system in the long term.

The rest of this paper is organized as follows. Section 2 presents a literature review. In Section 3 we present some general definitions and formulate the cogeneration and conventional models. Using backward dynamic programming we find the optimal solution that minimizes the total expected sum of planning and operation cogeneration costs. Then, we identify which model is preferable for the organization. Section 4 presents numerical examples for our analytical results. In particular, we show how model parameter values affect the choice of which model is preferable. Section 5 concludes.

## 2. Literature review

Researchers have investigated onsite cogeneration from technical, economic and environmental perspectives. Academic studies have introduced optimization methodologies for capacity planning, operational planning and combinations thereof. Most capacity-planning papers discuss optimization techniques that take into account site demands and that use various automatic methods. Studies on operation planning deal with operational system timing in accordance with economic constraints.

Ref. [5] proposes a techno-economic model of cogeneration for offices or buildings in Korea. Their model includes variable costs such as NG and electricity procurement from the grid with demand

constraints. The model further takes into account equipment capacity cost. To derive the optimal solution, the researchers use a mixed-integer linear programming technique and a branch and bound algorithm. They find that adoption of a cogeneration system is not economically viable for office buildings but is feasible for hotels. However, their paper does not make a comparison between adoption of a cogeneration system and use of conventional steam boiler. In addition, the model does not take into account the option of exporting spare electricity to the grid, whereas our model does.

Ref. [6] presents an energy dispatch algorithm that minimizes the total cost of running a combined heat and power system over a time horizon, in order to satisfy the total energy demand. Their model takes into account the cost of electricity obtained from the grid and additional operational costs such as fuel costs. It further incorporates the possibility of exporting surplus electricity to the grid (as in our case). However, their model does not include planning aspects and grid usage costs. The authors use a linear programming technique to analyze the model and obtain an optimal solution.

Ref. [7] proposes an optimal unit sizing method for cogeneration systems, using a numerical study to explore how the optimal solution is influenced by the uncertainty of energy demand. Their model's aim is to minimize the expected annual total cost, taking into account the equipment capacity, energy flow rate and sampling vector. The authors find that the optimal capacity of the fuel cell unit decreases if the cogeneration system is designed such that it takes the uncertainties into account. However, their numerical study examines only three representative days in a one-year period, and they ignore the influence of operational methodology.

Ref. [8] introduces a global and regional emission impact evaluation of distributed cogeneration systems with partial-load models. Taking into consideration the facility's efficiency and emission factor, the authors introduce the expected NO<sub>x</sub> and CO emission level, locally and globally, in accordance with cogeneration system load values. They found that the NO<sub>x</sub> rate increased while turbine load was low. However, the CO emission rate was high in every load.

Ref. [9] introduces the potential from implementing cogeneration in the plywood industry in India. The researchers use an “Annualized Life Cycle Cost” technique for investigating the most economic method of generating electricity and useful thermal energy. It was found that CHP with a steam turbine increases annual savings in operational energy.

Ref. [1] carries out a survey to obtain information regarding Jordan's energy consumption in the tourist accommodation sector and provide recommendations based on the results. The results show high willingness of hotels to reduce energy consumption by using efficient appliances. The authors suggest installing waste heat recovery systems in order to reduce diesel consumption.

Ref. [10] claims that combining investments in energy-efficient technologies with the promotion of good energy management practices can improve energy efficiency in the economy. Focusing on the European context, the authors argue that inclusion of energy management components in future energy policy will play an important role in Europe's ability to meet its energy efficiency targets for 2020, and later for 2050. Further research is suggested, quantifying the extent of the extended energy efficiency gap.

Ref. [11] reviews recent developments in technologies for waste heat recovery of exhaust gas from internal combustion engines. They discuss the potential energy savings and performances of organic Rankine cycle technologies. The authors find that using waste-to-heat technology can reduce emissions of greenhouse gas, sulfur oxides and nitrogen oxides.

Ref. [12] seeks to provide a better understanding of the processes applied in energy efficiency investment decisions. The author indicates that financial decision makers typically use

payback analysis to screen investments for risk; projects that pass the initial screening are subsequently evaluated with more traditional financial analysis methods. The author recommends incorporating risk management energy efficiency analysis in state and federal utility incentive programs and in government energy efficiency projects, suggesting that such an approach can create a sounder financial basis for efficiency investment decisions, reduce energy use, increase cash flows and contribute to environmental goals.

The trigeneration process is a cogeneration improvement that integrates the production of both heating and cooling from waste heat energy produced during electricity generation, as considered in [13–16]. Our paper does not deal with trigeneration.

As discussed above, the main innovation of the current paper is in dealing with an organization's dilemma as to whether or not adopt a cogeneration model. In addition, in contrast to models currently available in the literature, we suggest using the same heat exchanger for the two states of the cogeneration system (i.e., on and off). This approach is important due to the high cost of a single heat exchanger and is based on the fact that several firms, such as CAIN, APPROVIS and THERMAX, supply heat exchangers featuring heat recovery steam generators that contain two different connections. One is fed by the turbine's exhaust (corresponding to the situation when the system is on), while the other is served by the gas supply (usable when the system is off). Finally, the cogeneration system is planned such that the maximal electricity production rate is set in accordance with economic as well as demand considerations.

### 3. Solutions of the two models and comparison

#### 3.1. General data

Both the cogeneration model and the conventional model have to satisfy periodic demands for electricity and steam. We assume that there are  $I$  seasons,  $J$  days in each season and  $K$  possible different tariffs on each day. We use the term “period  $ijk$ ” to refer to the period in day  $j$  in season  $i$  during which the electricity tariff is  $k$ . The duration of this period is denoted by  $T_{ijk}$ . The electricity demand in period  $ijk$  is  $d_{ijk}^{(1)}$  with realization  $D_{ijk}^{(1)}$ , probability density function (pdf)  $f_{ijk}^{(1)}(\cdot)$  and cumulative distribution function (cdf)  $F_{ijk}^{(1)}(\cdot)$ . Similarly, the periodic steam demand is  $d_{ijk}^{(2)}$  with realization  $D_{ijk}^{(2)}$ , with pdf and cdf of  $f_{ijk}^{(2)}(\cdot)$  and  $F_{ijk}^{(2)}(\cdot)$ , respectively. Many firms (factories, hotels, hospitals etc.) need electricity and steam for different purposes. For instance, a factory may need electricity for illumination, cooling and computerization, on the one hand, and steam for producing a particular product, on the other hand. The need for heating is different during diverse seasons, while the demand for the product does not necessarily depend on the actual season. Thus, the two periodic demands are not correlated and assumed to be independent. We assume that  $F_{ijk}^{(1)}$  and  $F_{ijk}^{(2)}$  are invertible in the possible ranges of the demands. The decisions are determined according to the periodic electricity tariffs  $\{H_{ijk}\}$ . The electricity grid tariffs might change over long periods of time. Thus, we assume that these tariffs constitute average costs.

#### 3.2. The cogeneration model

##### 3.2.1. Model description

The cogeneration model consists of two main steps. The first step is the system setup, including capacity planning. In this step, the decision maker determines the value of  $P$ , the maximal quantity of electricity that the system produces per hour; the turbine cost per unit of electricity production rate is  $Q$ . Production of one

unit of electricity by the cogeneration system yields  $\Gamma$  units of derived steam. Consequently, the maximum quantity of generated steam is  $\Gamma P$  per hour. The heat-exchanger cost per one steam-ton per hour is  $V$ . In addition, infrastructure, regulation and installation costs, denoted by  $U_{co}$ ,  $F_{co}$  and  $O_{co}$  respectively, are required, and government subsidy  $E_{co}$  is possible. The capacity planning is performed in advance, before the exact demands are realized, in order to minimize the total expected cost-to-go, i.e., the cumulative planning and operation costs.

The second step is operation planning. At the beginning of period  $ijk$ , the realizations  $D_{ijk}^{(1)}$  and  $D_{ijk}^{(2)}$  are approximately known. At this point in time a binary decision, whether to turn the system on or off during that period, is made. If the system is on, then the electricity is produced by NG. The cost of producing one kW/h comprises the NG cost  $A$ , the cogeneration system maintenance cost  $W_{co}$  and electricity usage cost for the period,  $G_{ijk}$ . In this step, the decision maker must determine an electricity production rate for the period,  $X_{ijk}$ , that at least satisfies the steam demand in that period. Thus, in period  $ijk$ , the total quantity of electricity produced through the NG cogeneration system is equal to  $X_{ijk}T_{ijk}$ . If this quantity does not satisfy the demand for electricity, then the rest is purchased from the grid (with unit cost of  $H_{ijk}$ ). Otherwise, the surplus is sold to the grid, and the revenue per unit is  $C_{ijk}$ . We assume that this quantity is fixed and lower than the production cost, i.e.,

$$C_{ijk} \leq A + W_{co} + G_{ijk}, \quad (1)$$

such that there is no interest in producing extra electricity for selling. On the other hand, if the system is off during period  $ijk$ , then all electricity used in the period is purchased from the grid, and steam is produced by directly fueling the heat exchanger with NG. This process costs  $R$  for one unit of steam. While the system is off, the quantities of purchased electricity and produced steam are determined according to the realized demands.

Since surpluses of electricity and steam cannot be “stored” for later periods, then the decisions of the current period do not affect subsequent periods. Thus, the objective function at the beginning of period  $ijk$  is the (deterministic) period-specific cost.

The two steps of the cogeneration model problem are calculated by backwards dynamic programming. We begin with the operation planning.

##### 3.2.2. Solution for the cogeneration operation planning

Let  $ijk$  be the current period. The exact realizations  $D_{ijk}^{(1)}$  and  $D_{ijk}^{(2)}$  are known at the beginning of that period. If the system is set to be on, then the overall cost incurred in the period is equal to

$$\begin{aligned} Z_{ijk}^{\text{on}}(X_{ijk}, D_{ijk}^{(1)}, D_{ijk}^{(2)}) &= (A + W_{co} + G_{ijk})T_{ijk}X_{ijk} \\ &\quad + H_{ijk}(D_{ijk}^{(1)} - T_{ijk}X_{ijk})^+ \\ &\quad - C_{ijk}(T_{ijk}X_{ijk} - D_{ijk}^{(1)})^+, \end{aligned} \quad (2)$$

where  $(y)^+ = \max\{y, 0\}$  for any value  $y$ . Otherwise, the cost is

$$Z_{ijk}^{\text{off}}(D_{ijk}^{(1)}, D_{ijk}^{(2)}) = H_{ijk}D_{ijk}^{(1)} + RD_{ijk}^{(2)}. \quad (3)$$

We solve the following problem for the current period:

$$\min_{X_{ijk}} [g_{ijk}(X_{ijk}, D_{ijk}^{(1)}, D_{ijk}^{(2)}) := b_{ijk}Z_{ijk}^{\text{on}} + (1 - b_{ijk})Z_{ijk}^{\text{off}}] \quad (4)$$

$$\text{s.t. } 0 \leq X_{ijk} \leq P \quad (5)$$

$$b_{ijk} \in \{0, 1\}$$

$$\frac{D_{ijk}^{(2)}}{T_{ijk}\Gamma}b_{ijk} \leq X_{ijk} \leq M \cdot b_{ijk}. \quad (6)$$

Here  $b_{ijk}$  denotes the binary variable indicating whether the system is on ( $b_{ijk} = 1$ ) or off ( $b_{ijk} = 0$ ). For extremely large values of  $M$ , constraint (6) means that if  $b_{ijk}$  is equal to zero, then  $X_{ijk}$  necessarily vanishes. Otherwise,  $X_{ijk}$  must be greater than or equal to the value  $D_{ijk}^{(2)}/T_{ijk}\Gamma$ , which ensures that the period's steam demand is satisfied. By substituting the optimal value  $X_{ijk}^*$  in the objective function in (4), we obtain the Bellman function of the current period (see [17]):

$$B_{ijk}(P, D_{ijk}^{(1)}, D_{ijk}^{(2)}) = g_{ijk}(X_{ijk}^*(P, D_{ijk}^{(1)}, D_{ijk}^{(2)}), D_{ijk}^{(1)}, D_{ijk}^{(2)}). \quad (7)$$

The optimal solutions depend (among others) on the NG cost  $R$ , on the period-specific electricity tariff  $H_{ijk}$ , on the maximal cogeneration electricity production rate capacity  $P$  and on the realized demands in the current period. As we show below, there are four possibilities for the optimal value  $X_{ijk}^*$  and the derived Bellman function. The first one is

$$\begin{cases} X_{ijk}^* = 0 \\ B_{ijk}(P, D_{ijk}^{(1)}, D_{ijk}^{(2)}) = H_{ijk} \cdot D_{ijk}^{(1)} + R_{ijk} \cdot D_{ijk}^{(2)}. \end{cases} \quad (8)$$

In this case, the system is off, all the electricity is purchased from the grid, and all the steam is produced by directing the NG toward the heat exchanger (i.e., using the heat exchanger bypass). The second possibility is

$$\begin{cases} X_{ijk}^* = \frac{D_{ijk}^{(2)}}{T_{ijk}\Gamma} \\ B_{ijk}(P, D_{ijk}^{(1)}, D_{ijk}^{(2)}) = H_{ijk} \cdot D_{ijk}^{(1)} + \frac{A+W_{co}+G_{ijk}-H_{ijk}}{\Gamma} D_{ijk}^{(2)}, \end{cases} \quad (9)$$

namely, the system is on, and the quantity of electricity produced by cogeneration is such that the actual steam demand is exactly satisfied (note that the maximal rate  $P$  is set such that it is necessarily possible to choose  $X_{ijk}^*$  according to (9); see constraint (6) as well as Section 3.2.3. below). The quantity of electricity produced does not fully satisfy the demand for electricity, and the rest is purchased from the grid. The third case is

$$\begin{cases} X_{ijk}^* = \frac{D_{ijk}^{(2)}}{T_{ijk}\Gamma} \\ B_{ijk}(P, D_{ijk}^{(1)}, D_{ijk}^{(2)}) = C_{ijk} \cdot D_{ijk}^{(1)} + \frac{A+W_{co}+G_{ijk}-C_{ijk}}{\Gamma} D_{ijk}^{(2)}. \end{cases} \quad (10)$$

As in the previous case, the quantity of electricity produced by the cogeneration system meets the exact steam demand, but the quantity of electricity produced exceeds the electricity demand, so the extra electricity is sold to the grid. The last possible situation is

$$\begin{cases} X_{ijk}^* = \min\left\{\frac{D_{ijk}^{(1)}}{T_{ijk}}, P\right\} \\ B_{ijk}(P, D_{ijk}^{(1)}, D_{ijk}^{(2)}) = (A+W_{co}+G_{ijk})\min\{D_{ijk}^{(1)}, P \cdot T_{ijk}\} + (D_{ijk}^{(1)} - P \cdot T_{ijk})^+ H_{ijk}. \end{cases} \quad (11)$$

In this case, the cogeneration system produces a quantity of electricity that satisfies the steam demand. This does not fully cover the demand for electricity, and it is better to keep producing the rest of required electricity by the system. However, the total capacity of electricity that can be produced in this way (i.e.,  $P \cdot T_{ijk}$ ) may be lower than the demand, so in this case, the system produces the maximal possible capacity of electricity, and the rest is purchased from the grid.

We now examine how the optimal solution is affected by the parameter values. We consider two ranges (low and high) for  $R$ :

$$\begin{cases} R \leq \frac{A+W_{co}+G_{ijk}-C_{ijk}}{\Gamma} \\ R > \frac{A+W_{co}+G_{ijk}-C_{ijk}}{\Gamma}. \end{cases} \quad (12)$$

Given the value of  $R$ , we consider three ranges (low, medium and high) for  $H_{ijk}$ :

$$\begin{cases} H_{ijk} < A + W_{co} + G_{ijk} - \Gamma R \\ A + W_{co} + G_{ijk} - \Gamma R \leq H_{ijk} \leq A + W_{co} + G_{ijk} \\ H_{ijk} > A + W_{co} + G_{ijk}. \end{cases} \quad (13)$$

The optimal solutions are described in Propositions 1–6, whose proofs are based on basic algebra. We first assume that the NG cost is low such that the first term of (12) holds. If the electricity tariff is low as well, then it is always better not to turn the system on. This is summarized in the following proposition.

**Proposition 1.** *If the first terms of both (12) and (13) are satisfied, then the optimal solution is (8), for any values of  $D_{ijk}^{(1)}$  and  $D_{ijk}^{(2)}$ . □*

According to Proposition 1, if the NG and electricity costs are both low, it is always better to purchase all electricity from the grid, and to produce all steam directly from the NG. Using the cogeneration system to produce electricity necessarily costs more than purchasing it from the grid, regardless of the period-specific values of electricity and steam demand. As shown below, these demand values do affect the optimal solutions in the other cases.

Now let the electricity tariff be medium; then the optimal solution is according to the following proposition.

**Proposition 2.** *If the first term of (12) and the second term of (13) are met, then the optimal solution depends on the ratio between  $D_{ijk}^{(2)}$  and  $D_{ijk}^{(1)}$  as follows.*

If

$$D_{ijk}^{(2)} \leq \Gamma D_{ijk}^{(1)}, \quad (14)$$

then the optimal solution is (9).

If

$$\Gamma D_{ijk}^{(1)} < D_{ijk}^{(2)} \leq \frac{(H_{ijk} - C_{ijk})\Gamma}{A + W_{co} + G_{ijk} - C_{ijk} - \Gamma R} D_{ijk}^{(1)}, \quad (15)$$

then the solution becomes (10).

If

$$D_{ijk}^{(2)} > \frac{(H_{ijk} - C_{ijk})\Gamma}{A + W_{co} + G_{ijk} - C_{ijk} - \Gamma R} D_{ijk}^{(1)}, \quad (16)$$

then (8) is optimal. □

Condition (14) means that if the system produces electricity such that the derived quantity of steam exactly satisfies the steam demand, then the quantity of electricity produced is not enough to fully meet the demand for electricity. In this case, it is optimal to produce a quantity of electricity that satisfies the steam demand  $D_{ijk}^{(2)}$  and to purchase the rest of the electricity from the grid. Next, if (14) is not satisfied, then if the system produces the same capacity of electricity, then it exceeds the electricity demand  $D_{ijk}^{(1)}$ . The extra electricity can then be sold. According to assumption (1), the revenue from selling the electricity is lower than the cost of production, which leads to some loss. As a result, if the quantity of extra electricity produced by the system is not too high, such that (15) is met, then the savings obtained by avoidance of the tariff and NG costs is still higher than the loss. Thus, it is still better to turn the system on and sell the extra electricity. On the other hand, if  $D_{ijk}^{(2)}$  exceeds a critical value according to (16), then the quantity of spare electricity becomes high, such that the loss becomes significant. In this case, it is better to turn the system off and pay the  $H_{ijk}$  and  $R$  unit costs.

If the electricity tariff becomes high, then it may be optimal to produce quantities of electricity beyond the quantity that exactly satisfies the steam demand (according to the electricity demand). Moreover, the optimal solution may be affected by the maximal electricity production rate  $P$ . The following proposition refers to the case of high tariff (and low NG cost).



**Proposition 3.** *If the first term of (12) and the third term of (13) hold, then the optimal solution is as follows.*

If (14) is satisfied, then the optimal solution becomes (11). Under (15) and (16), solutions (10) and (8) remain optimal, respectively.  $\square$

The difference between Propositions 2 and 3 is manifested in the case of a low ratio between the period-specific demands for steam and electricity (i.e., (14) is met). In this case, producing electricity such that the derived quantity of steam is equal to the steam demand does not satisfy the electricity demand. Under a medium tariff (the second term of (13)), it is better to purchase the rest of electricity from the grid, as the first part of Proposition 2 indicates. On the other hand, if the tariff becomes high (according to the third term of (13), as assumed in the first part of Proposition 3), then it is better to use the cogeneration system to fulfill the remaining electricity demand. Therefore, if possible (according to constraint (5)), the system produces the entire required quantity  $D_{ijk}^{(1)}$ . Otherwise, the maximal quantity  $P \cdot T_{ijk}$  is produced, and the rest is purchased from the grid (see (11)).

Next we consider the case of high NG cost, such that the second term of (12) is satisfied. If the electricity tariff is low, then we have the following proposition.

**Proposition 4.** *If the second term of (12) holds, as well as the first term of (13), then the optimal solution is as follows.*

If the steam demand satisfies (14) or (15), then the optimal solution is (8). If (16) is satisfied, then solution (10) is valid.  $\square$

Contrary to Proposition 1, since the NG cost is now higher, then it may be preferable to turn the system on. Moreover, we find the following result. If the NG cost is low and the electricity tariff is medium or high, as assumed in Proposition 2 and 3, then it is better to turn the system on while the steam demand is low compared to the electricity demand. On the other hand, if the NG is high and the electricity tariff is low (as assumed in Proposition 4), then the system should be turned on while the steam demand is high relative to the electricity demand.

If the NG cost remains high and the tariff increases, it is always preferable to turn the system on and produce electricity in order to satisfy the steam demand at least, as summarized in the next two propositions.

**Proposition 5.** *Let the second terms of (12) and (13) hold; then the system should always be turned on as follows. If the steam demand satisfies (14), then the optimal solution is (9). Otherwise, the solution is (10).  $\square$*

**Proposition 6.** *Let the second term of (12) and the third of (13) hold. If the steam demand satisfies (14), then the optimal solution is (11). Otherwise, the solution is (10).  $\square$*

As noted above, if (14) is satisfied, then additional electricity is required beyond the electricity produced to fulfill the steam demand. Furthermore, under a medium tariff (Proposition 5), the rest of the electricity should be purchased from the grid, whereas under a high tariff (Proposition 6) it is preferable to use the system to fulfill the remainder of the electricity demand (subject to constraint (5)). Under a high NG cost, in contrast to Propositions 2 and 3 (corresponding to low NG cost), solution (10) remains optimal for every  $D_{ijk}^{(2)}$  that does not meet (14), even if  $D_{ijk}^{(2)}$  increases.

That is, it is necessarily preferable to use the system to produce extra electricity (up to the quantity that covers the steam demand) and sell it to the grid rather than to turn the system off and pay the high cost  $R$ .

We now proceed to the stochastic part of the problem: solving the capacity planning step.

### 3.2.3. Solution for the cogeneration capacity planning

Given the cogeneration system's maximal electricity production rate  $P$ , the setup cost associated with implementation of the cogeneration system is equal to

$$z_{co}(P) = [Q + V\Gamma]P + U_{co} + F_{co} + O_{co} - E_{co}. \quad (17)$$

The goal in this step is to select a value of  $P$  that minimizes the expected total cost-to-go, i.e., the cumulative setup and operation costs. Let  $N_{ij}$  be the number of days of type  $j$  in season  $i$  in a single year, and assume that the process is planned for  $Y$  years ( $Y$  is also referred to as the "process duration"). The objective function is obtained by the Bellman formula as follows:

$$Z_{co}(P) = z_{co}(P) + Y \sum_{i=1}^I \sum_{j=1}^J N_{ij} \sum_{k=1}^K E \left[ B_{ijk} \left( P, d_{ijk}^{(1)}, d_{ijk}^{(2)} \right) \right]. \quad (18)$$

The second term of (18) denotes the expected total operation cost (during all the periods). The cogeneration system's maximal steam production rate is  $\Gamma P$ . In order to ensure that steam demand is satisfied across all periods, the value of  $P$  must fulfill

$$\Pr \left( d_{ijk}^{(2)} > \Gamma T_{ijk} P \right) \leq \varepsilon \quad (19)$$

for a small value of  $\varepsilon$ , for every  $i, j$  and  $k$  (here the symbol  $\Pr(\cdot)$  denotes the probability of an event). Equivalently,  $P$  has to satisfy the criterion of the following remark.

**Remark 1.** Constraint (19) is satisfied if and only if  $P \geq \frac{1}{\Gamma T_{ijk}} \left( F_{ijk}^{(2)} \right)^{-1} (1 - \varepsilon)$  (recall that by our assumption in Subsection 3.1., the inverse functions  $\{ (F_{ijk}^{(2)})^{-1} \}$  are well defined in the open interval  $(0, 1)$ ).  $\square$

Based on Remark 1, let

$$P_0 = \max_{i,j,k} \left\{ \frac{1}{\Gamma T_{ijk}} \left( F_{ijk}^{(2)} \right)^{-1} (1 - \varepsilon) \right\}, \quad (20)$$

then the following problem has to be solved

$$\min_{P \geq P_0} Z_{co}(P). \quad (21)$$

The possible expected values in the second term of the right-hand side of (18), are obtained from Propositions 1–6 by using the formula

$$E \left[ B_{ijk} \left( P, d_{ijk}^{(1)}, d_{ijk}^{(2)} \right) \right] = \int_0^\infty \left[ \int_0^\infty B_{ijk} \left( P, D_{ijk}^{(1)}, D_{ijk}^{(2)} \right) f_{ijk}^{(2)} \left( D_{ijk}^{(2)} \right) dD_{ijk}^{(2)} \right] f_{ijk}^{(1)} \left( D_{ijk}^{(1)} \right) dD_{ijk}^{(1)}. \quad (22)$$

According to Propositions 1–6, the expected operational cost in period  $ijk$  depends on  $P$  if and only if the tariff  $H_{ijk}$  is higher than  $A + W_{co} + G_{ijk}$  (assuming that  $P \geq P_0$ ). Differentiating (22) twice with respect to  $P$  in this case leads to

$$\frac{dE \left[ B_{ijk} \left( P, d_{ijk}^{(1)}, d_{ijk}^{(2)} \right) \right]}{dP} = - (H_{ijk} - (A + W_{co} + G_{ijk})) T_{ijk} \left( 1 - F_{ijk}^{(1)}(T_{ijk}P) \right)$$

as well as

$$\frac{d^2 E \left[ B_{ijk} \left( P, d_{ijk}^{(1)}, d_{ijk}^{(2)} \right) \right]}{dP^2} = (H_{ijk} - (A + W_{co} + G_{ijk})) T_{ijk}^2 f_{ijk}^{(1)}(T_{ijk}P) \geq 0.$$

From this result as well as by the linearity of (17) in  $P$  we obtain the following remark.

**Remark 2.** The objective function (18) is convex, and its derivative is

$$\frac{dZ_{co}(P)}{dP} = Q + \Gamma V - Y \sum_{i=1}^I \sum_{j=1}^J N_{ij} \sum_{k=1}^K (H_{ijk} - (A + W_{co} + G_{ijk}))^+ \times T_{ijk} (1 - F_{ijk}^{(1)}(T_{ijk}P)). \quad \square \quad (23)$$

From (23) we find that  $\lim_{P \rightarrow \infty} \frac{dZ_{co}(P)}{dP} = Q + \Gamma V > 0$ , and therefore the optimal value, denoted by  $P^*$ , is finite, as follows.

**Proposition 7.** If the cogeneration system's minimal electricity production rate  $P_0$  satisfies

$$\sum_{i=1}^I \sum_{j=1}^J N_{ij} \sum_{k=1}^K (H_{ijk} - (A + W_{co} + G_{ijk}))^+ T_{ijk} (1 - F_{ijk}^{(1)}(T_{ijk}P_0)) \leq \frac{Q + \Gamma V}{Y}, \quad (24)$$

then  $P^* = P_0$ . Otherwise,  $P^* > P_0$  such that

$$\sum_{i=1}^I \sum_{j=1}^J N_{ij} \sum_{k=1}^K (H_{ijk} - (A + W_{co} + G_{ijk}))^+ T_{ijk} (1 - F_{ijk}^{(1)}(T_{ijk}P^*)) = \frac{Q + \Gamma V}{Y}. \quad (25)$$

The expected cost under the cogeneration model is  $Z_{co}(P^*)$  (see (18)).  $\square$

Satisfying the negation of (24) means that the derivative (23) is negative at  $P_0$ , and by the convexity it vanishes at some higher value  $P^*$ . Denote

$$Y_0 = \frac{Q + \Gamma V}{\sum_{i=1}^I \sum_{j=1}^J N_{ij} \sum_{k=1}^K (H_{ijk} - (A + W_{co} + G_{ijk}))^+ T_{ijk} (1 - F_{ijk}^{(1)}(T_{ijk}P_0))},$$

then from (24), once  $Y$  exceeds  $Y_0$ , the optimal value  $P^*$  becomes greater than  $P_0$ . This may occur when there are many periods in which the tariff  $H_{ijk}$  is higher than the cost  $A + W_{co} + G_{ijk}$  (or a few periods in which  $H_{ijk}$  is substantially greater than that cost). It can also occur in cases of high electricity demand or high period durations. Alternatively, if the turbine and heat-exchanger costs  $Q$  and  $V$  are low, then (24) may also not be met. In all those cases, turning the system on is supposed to be preferable. In particular, if the electricity demand is higher than the quantity that satisfies the steam demand, then it is better to use the system to fulfill the remaining electricity demand (or as much of it as possible). Thus, it is desirable to plan a maximal electricity production rate that exceeds  $P_0$  (the minimum electricity production rate that ensures that steam demands are met). In other cases, (24) is satisfied, and there is no need for the maximal electricity production rate to exceed  $P_0$ . The value of  $P_0$  is affected by the distribution of steam demand (see (20)). In particular, high variances lead to greater value, due to the constraint of satisfying all steam demands at all costs. We now proceed to formulate the conventional model.

### 3.3. The conventional model

We first formulate the conventional model, which is significantly less complicated than the cogeneration model.

The first step is the capacity planning. The maximal steam production rate  $L$  is given, such that a boiler conversion cost  $\Omega$  is incurred per unit. In addition, fixed infrastructure, regulation and installation costs, respectively denoted by  $U_{ec}$ ,  $F_{ec}$  and  $O_{ec}$ , are

required. As in the cogeneration model, government subsidy  $E_{ec}$  is possible. The setup cost is

$$Z_{ec}(L) = \Omega L + U_{ec} + F_{ec} + O_{ec} - E_{ec}. \quad (26)$$

In order to ensure that steam demand is satisfied in all periods, the maximal rate  $L$  has to be high enough, such that

$$\Pr(d_{ijk}^{(2)} > T_{ijk}L) \leq \varepsilon \quad (27)$$

for every  $i, j$  and  $k$ . Similarly to the cogeneration model, we obtain the following remark.

**Remark 3.** Constraint (27) is met if and only if  $L \geq \frac{1}{T_{ijk}} (F_{ijk}^{(2)})^{-1} (1 - \varepsilon)$ .  $\square$

Let  $L_0 = \max_{i,j,k} \left\{ \frac{1}{T_{ijk}} (F_{ijk}^{(2)})^{-1} (1 - \varepsilon) \right\}$ , then  $L$  must be set such that  $L \geq L_0$ .

In the operation planning step, the electricity is purchased from the grid, while the NG is made to flow to the boiler. The unit NG cost is  $a$ , and the conventional steam boiler's maintenance cost per unit of steam is  $W_{ec}$ .

As a result, the operational cost in period  $ijk$  (given the realized demands) is

$$Z_{ijk}^{ec}(D_{ijk}^{(1)}, D_{ijk}^{(2)}) = H_{ijk} D_{ijk}^{(1)} + (a + W_{ec}) D_{ijk}^{(2)}.$$

That cost is analogous to the cogeneration operation cost corresponding to the case in which the system is turned off (see (3)). In particular, the cost is not affected by the maximal steam production rate  $L$  (as long as  $L$  is greater than or equal to  $L_0$ ).

The expected total cost-to-go in the capacity planning step is

$$Z_{ec}(L) = z_{ec}(L) + Y \sum_{i=1}^I \sum_{j=1}^J N_{ij} \sum_{k=1}^K E \left[ Z_{ijk}^{ec}(d_{ijk}^{(1)}, d_{ijk}^{(2)}) \right], \quad (28)$$

and the following problem is obtained:

$$\min_{L \geq L_0} Z_{ec}(L). \quad (29)$$

The solution of (29) is trivial. The only component of the objective function (28) that depends on  $L$  is the setup cost  $z_{ec}$ , which, according to (26), is linearly increasing in  $L$ . Hence, the following proposition presents the solution.

**Proposition 8.** The optimal solution of (29) is  $L^* = L_0$ , and the expected total cost is

$$\Omega L_0 + U_{ec} + F_{ec} + O_{ec} - E_{ec} + Y \sum_{i=1}^I \sum_{j=1}^J N_{ij} \sum_{k=1}^K (H_{ijk} E[d_{ijk}^{(1)}] + (a + W_{ec}) E[d_{ijk}^{(2)}]). \quad \square$$

In the following subsection we compare between the cogeneration and the conventional models and determine which one is preferable.

### 3.4. Comparison and setting of the preferable model

Let

$$\text{ANN}_{co}(P^*) = \sum_{i=1}^I \sum_{j=1}^J N_{ij} \sum_{k=1}^K E \left[ B_{ijk}(P^*, d_{ijk}^{(1)}, d_{ijk}^{(2)}) \right]$$

and

$$\text{ANN}_{ec} = \sum_{i=1}^I \sum_{j=1}^J N_{ij} \sum_{k=1}^K (H_{ijk} E[d_{ijk}^{(1)}] + (a + W_{ec}) E[d_{ijk}^{(2)}])$$

be the annual expected operation costs corresponding to the cogeneration and conventional models, respectively. According to the previous calculations, we obtain the following theorem.

**Theorem 1.** Given the parameters of the problem, it is better to use the cogeneration model if and only if

$$z_{co}(P^*) + Y \cdot ANN_{co}(P^*) < z_{ec}(L_0) + Y \cdot ANN_{ec}. \quad (30)$$

Although the operation cost is expected to be lower under the cogeneration model than under the conventional one, the setup cost of the former is usually greater. As a result, the profitability of the cogeneration approach is significantly affected by the process duration  $Y$ , as follows. The optimal maximal electricity production rate depends on  $Y$ , such that  $P^*(Y) = P_0$  if  $Y$  is lower than or equal to  $Y_0$ . Moreover, Eq. (25) implicitly defines a function  $P^*(Y)$  for  $Y \in [Y_0, \infty)$ , which according to the Implicit Function Theorem, is increasing.

If the process duration  $Y$  grows, then  $z_{co}$  (which is increasing in  $P$ ) is increasing, while  $ANN_{co}$  (which is decreasing in  $P$ ) is decreasing. Assume that

$$\begin{cases} z_{co}(P_0) > z_{ec}(L_0) \\ ANN_{co}(P_0) < ANN_{ec}. \end{cases} \quad (31)$$

Since in real life electricity demands are bounded, we assume that  $P^*(Y)$  is bounded, and denote  $P^{\max} = \lim_{Y \rightarrow \infty} P^*(Y)$ .

If the process duration is small and tends to zero, then the LHS and RHS of (30) are, respectively  $z_{co}(P_0)$  and  $z_{ec}(L_0)$ , that is, inequality (30) is not satisfied. Thus, the conventional model is preferable. On the other hand, when  $Y$  grows and tends to infinity, (30) becomes approximately equivalent to

$$z_{co}(P^{\max}) - z_{ec}(L_0) < Y(ANN_{ec} - ANN_{co}(P^{\max})),$$

which is necessarily met. Hence, the cogeneration model becomes preferable. Our numerical calculations show that, as expected, the difference between the two sides of (30) is monotonically increasing in  $Y$ . Let ROI denote the return-on-investment time of the cogeneration model. Under our assumptions, ROI is the unique value that satisfies

$$z_{co}(P^*(ROI)) + ROI \cdot ANN_{co}(P^*(ROI)) = z_{ec}(L_0) + ROI \cdot ANN_{ec}.$$

**Corollary 1.** Under the assumptions of (31), for  $Y \leq ROI$ , the conventional model is preferable. Once  $Y$  exceeds ROI, the cogeneration model becomes preferable. In the latter case, the saving obtained through adoption of the cogeneration system (as compared with adoption of the conventional NG system) is equal to

$$S(Y) = \frac{Y(ANN_{ec} - ANN_{co}(P^*(Y))) - (z_{co}(P^*(Y)) - z_{ec}(L_0))}{z_{ec}(L_0) + Y \cdot ANN_{ec}}. \quad (32)$$

Our analytical results are demonstrated numerically in the following section.

#### 4. Numerical examples and sensitivity analysis

The examples in this section are based on the electricity tariffs of the Israel Electric Corporation. The Israeli electricity bill is composed of nine tariffs as follows. There are three kinds of seasons (i.e.,  $I = 3$ ): summer, winter and the intermediate seasons. In addition, there are three possible tariffs on each day ( $K = 3$ ): a high tariff for peak-usage rush hours, a low tariff for low-usage hours, and a medium tariff. The electric company also distinguishes between three types of days ( $J = 3$ ), based on Jewish religious practices, corresponding to different electricity and steam demands: business days (Sundays through Thursdays), Fridays/holiday eves, and Saturdays/holidays. For more information, see [18]. All the parameters are given in Table 1.

First, we assume that the electricity as well as steam demands are uniformly distributed, with means according to Table 1

(columns 4–5). Also, for this case we assume that demand realizations may range between 80% and 120% of those averages. Generally, the cogeneration operation cost is equal to  $A + W_{co} + G_{ijk}$  per one electricity unit, if the system is on, or to  $R$  per one steam unit, if the system is off. The basic values are given in Table 1, but bargaining may change the values of  $A$  and  $R$ . As a result, we consider a range of 20% under and over those values. The ratio is denoted by  $r$ , that is,  $r$  may lie in the interval  $[0.8, 1.2]$ . In particular,  $r = 1$  means that the parameters are the same as given in Table 1.

The times until return-on-investment is achieved (ROI) are presented in Fig. 1. Each of the three curves presents the ROI duration as a function of  $r$ , for a different electricity tariff level. Under the current grid tariffs (100%) and current cogeneration operation cost ( $r = 1$ ), ROI is equal to 5 years. Thus, if the process duration is 5 years or more, the cogeneration model is preferable. Otherwise, the conventional model is better. Decreasing the operation cost ( $r < 1$ ) reduces the ROI to 4 years, while increasing it ( $r > 1$ ) leads to higher ROIs of 6–8 years. The electricity grid tariffs affect ROI as follows. If the tariffs increase by 20% (see “120%” in Fig. 1), ROI is reduced to 4 years under the current operation cost. It can be reduced to 3 years if that cost is lower, and does not exceed 5 years even when that cost is higher. On the other hand, if the electricity tariffs decrease by 20% (see “80%”), ROI duration can increase significantly. In particular, it is equal to 8 years when  $r = 1$ .

Assuming that the cogeneration model is adopted, Fig. 2 presents the percentage of time during which the system is supposed to be turned on. Under the current tariffs and operation cost, the system is supposed to be turned on for 75% of time. This number may increase to 100% if the operation cost decreases, or decrease if the operation cost increases. Nevertheless, the system is on more than 50% of the time. If the electricity tariffs increase, then the system is supposed to be on almost 100% of the time. On the other hand, if the tariffs decrease, the system is supposed to be off most of the time, unless the operation cost is reduced. However, as ROI is usually long under those tariffs, the cogeneration model is not adopted unless the process duration is long.

Fig. 3 illustrates the expected savings associated with the cogeneration model, as a function of the process duration  $Y$ . Under

**Table 1**  
Data for cogeneration and conventional models.

$ijk$	$H_{ijk}$	$T_{ijk}$	$d_{ijk}^{(1)}$	$d_{ijk}^{(2)}$	$N_{ij}$	Additional parameters
111	0.358	10	68	36	44	$G_{ijk} = 0.03$ ( $\forall i, j, k$ )
112	0.5614	7	48	25	44	$C_{ijk} = 0.3$ ( $\forall i, j, k$ )
113	1.3707	7	48	25	44	$A = 0.45$
121	0.358	24	82	43	9	$W_{co} = 0.06$
122	0.5614	0	0	0	9	$R = 0.2$
123	1.3707	0	0	0	9	$\Gamma = 0.96$
131	0.358	24	41	22	9	$Q = 3500$
132	0.5614	0	0	0	9	$V = 350$
133	1.3707	0	0	0	9	$U_{co} = 150,000$
211	0.402	16	77	79	65	$F_{co} = 150,000$
212	0.7295	2	9.7	9.8	65	$O_{co} = 150,000$
213	1.2598	6	29	30	65	$E_{co} = 350,000$
221	0.402	20	48	49	13	$a = 0.22$
222	0.7295	4	9.7	9.8	13	$W_{ec} = 0.06$
223	1.2598	0	0	0	13	$\Omega = 1300$
231	0.402	20	24	25	12	$U_{ec} = 110,000$
232	0.7295	2	2.4	2.5	12	$F_{ec} = 110,000$
233	1.2598	2	2.4	2.5	12	$O_{ec} = 110,000$
311	0.348	8	43	51	139	$E_{ec} = 250,000$
312	0.4403	2	11	13	139	$\varepsilon = 0.01$
313	0.5532	14	75	90	139	
321	0.348	14	37	45	35	
322	0.4403	10	27	32	35	
323	0.5532	0	0	0	35	
331	0.348	20	27	33	39	
332	0.4403	4	5.3	6.5	39	
333	0.5532	0	0	0	39	

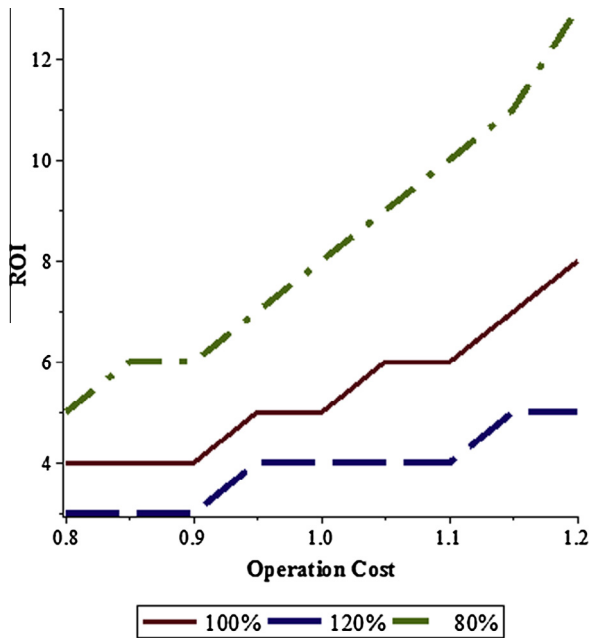


Fig. 1. The effect of electricity tariffs on ROI.

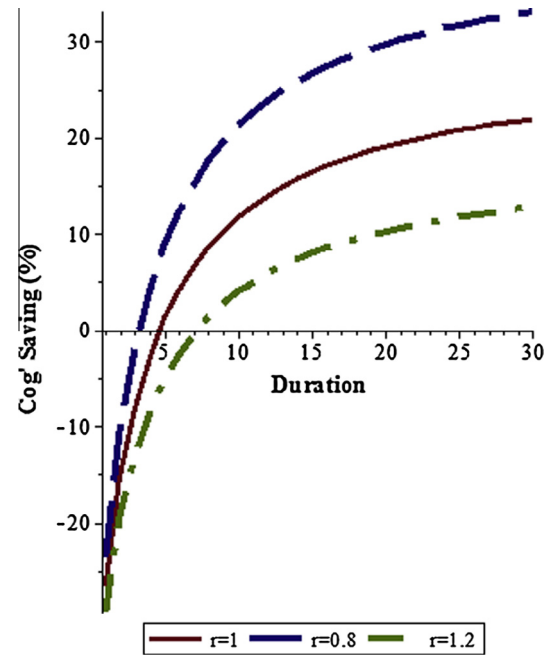


Fig. 3. Cogeneration savings.

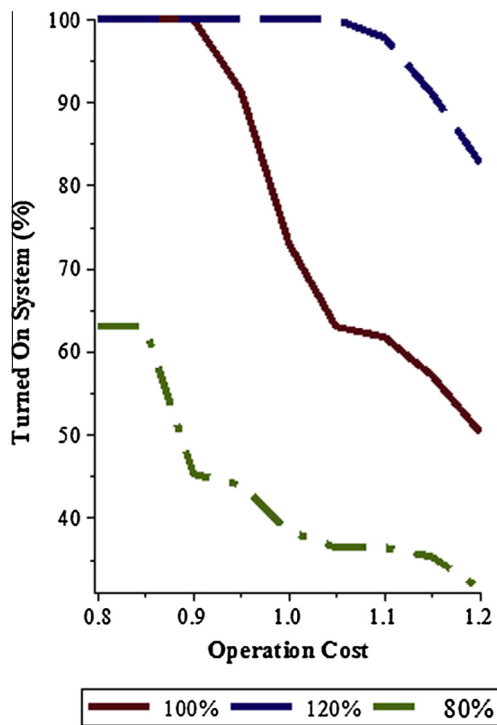
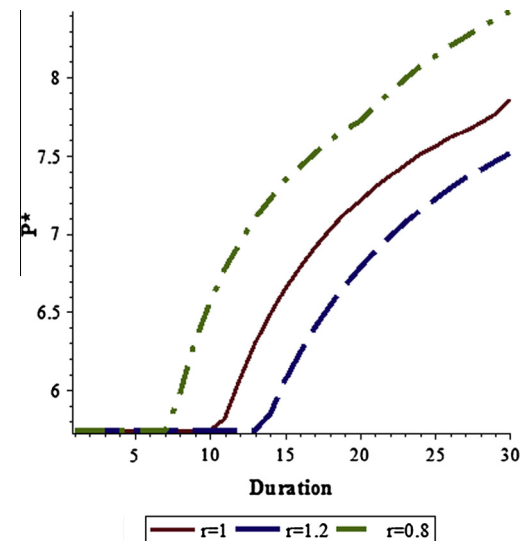


Fig. 2. The effect of electricity tariffs on the system.

Fig. 4. The optimal values  $P^*(Y)$ : high (low) electricity (steam) demands.

the current operation cost, savings are low if  $Y=5$ , but they increase to 10% when  $Y$  increases to 8 years. When  $Y$  exceeds 13 years, the cogeneration model is associated with cost savings of at least 15%. The cogeneration savings become more substantial if the operation cost is low: for a 5-year process duration savings are close to 10%, and for long processes savings may exceed 30%. On the other hand, if the process duration is shorter than or equal to 3 years, then the cogeneration model is not efficient and may be 20% more costly than the conventional model.

Next, we consider the optimal maximal electricity production rate  $P^*$ . For the data in Table 1, the value of  $P_0$  is 8.1. Increasing

(decreasing) the possible range of steam demand leads to greater (lower) values of  $P_0$ . Our sensitivity analysis indicates that the optimal value  $P^*$  is almost always equal to  $P_0$  (if the process duration is very long and exceeds 20 years, the optimal value may increase slightly). On the other hand, if the expected values and the variances of electricity demands increase simultaneously, while those of the steam demands decrease, then the optimal value  $P^*$  may significantly exceed  $P_0$ , as shown in Fig. 4 (here  $P_0 = 5.74$ ).

Assuming that the periodic demands for electricity and steam are normally distributed, let us now consider how the distribution parameters affect our results. As Table 1 indicates, the cogeneration operation costs are fixed. We first show how the means of the demand affect results. If both electricity and steam demand means are changed, compared with the values in Table 1, then the ROI of the cogeneration, as well as the cogeneration savings may significantly be changed, as shown in Fig. 5. If the demand means grow fivefold, then cogeneration may save more than 10%



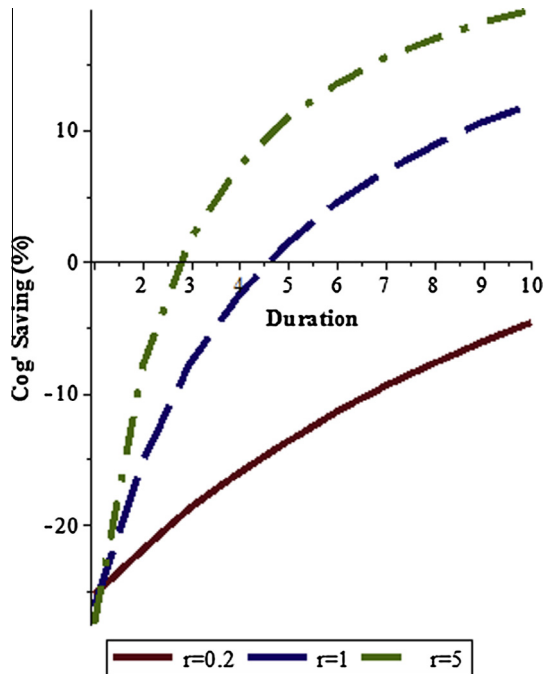


Fig. 5. Effects of expected demands on cog' saving.

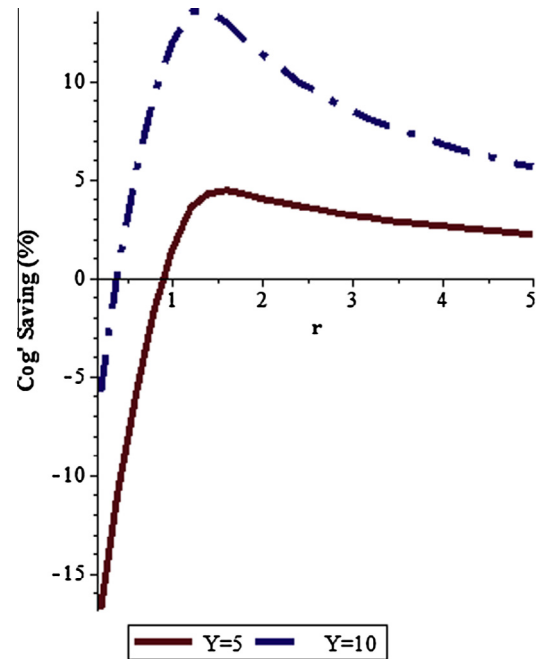


Fig. 7. Cog' saving for different electricity demand means.

in a 5-year period and almost 20% over 10 years (see " $r = 5$ "). On the other hand, if these means are reduced fivefold, then the conventional model is preferable even for 10 years (see " $r = 0.2$ "). If the means of only one type of demand are changed, then the results are not so clear and non-trivial, as shown in Figs. 6–8. If the electricity demand means grow (while the steam demands remain the same), then the cogeneration saving is reduced for long processes, 6 years or more (see Fig. 6). Otherwise (4–5 years), they negligibly increase.

According to Fig. 7, if the project duration is 10 years (" $Y = 10$ "), then the most significant saving is obtained while the electricity

demand means remain the same as in Table 1 (i.e.  $r = 1$ ). These savings are reduced if the means either grow or decrease. These results are even clearer if the steam demand means are changed (while the electricity demands remain the same), as Fig. 8 presents. Assume that  $Y = 10$ . If the steam demands are the same ( $r = 1$ ), then the cogeneration saving is equal to 12%. If the steam demand means either a fivefold increase or fivefold decrease, then the savings are significantly reduced (less than 3%). If  $Y = 5$ , then the cogeneration leads to negligible savings under the current means, but may significantly increase costs if these means either grow or decrease.

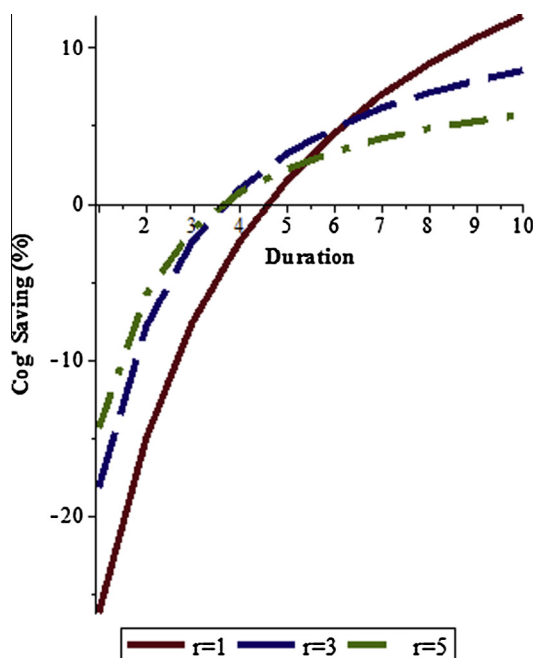


Fig. 6. Effects of expected electricity demands on cog' saving.

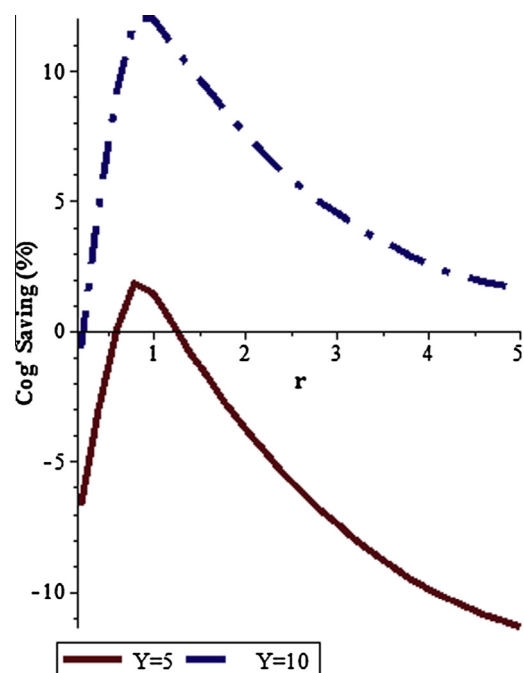


Fig. 8. Cog' saving for different steam demand means.

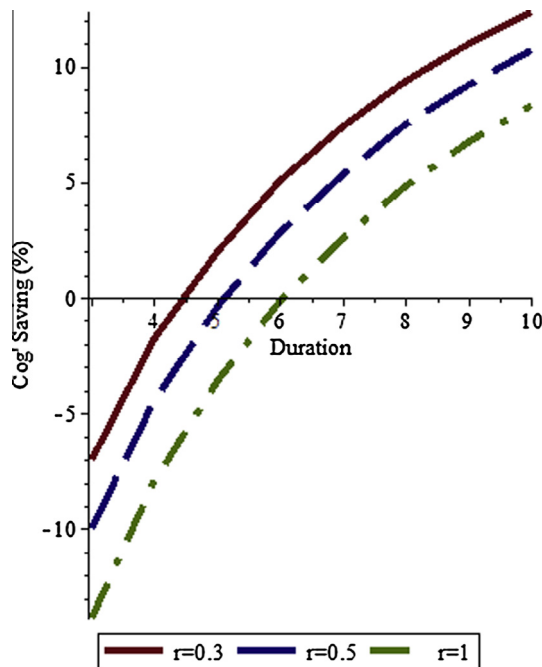


Fig. 9. Cog' saving for different steam demand SD.

Finally, we consider how changing the possible range lengths of the demands (namely, their standard-deviations) may affect our results. We find the following non-trivial result: the greater the steam variance, the smaller the cogeneration saving (or higher ROI), as shown in Figs. 9 and 10. The parameter  $r$  now denotes the possible range of the steam demand (under normal distribution). If  $r=0.2$ , then the demand may increase or decrease by 20% from its mean, as we have assumed so far. If  $r=1$ , the demand may be between 0 and twice the mean. When  $r$  tends to 0, the demands become deterministic.

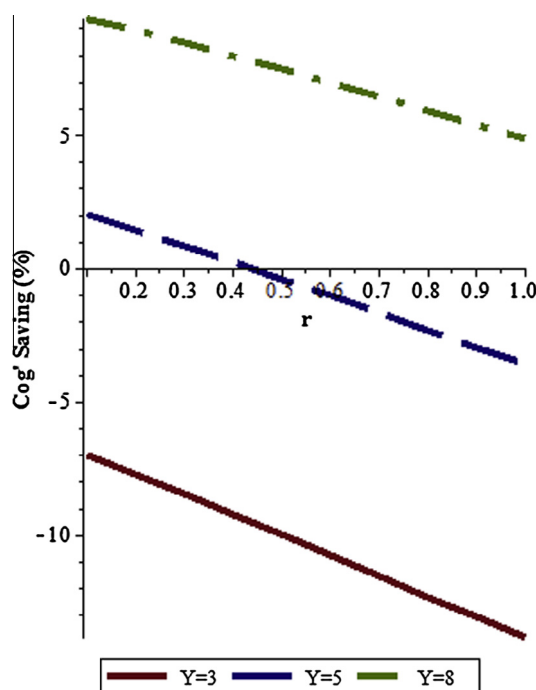


Fig. 10. Effects of steam demand SD on Cog' saving.

If  $Y=8$ , then the cogeneration may save up to 10% if the standard deviation tends to zero and 5% if is changed by 100%. Assuming that  $Y=5$ , if the steam demands' possible ranges are lower than 40% of the means (in each direction), then the cogeneration reduces the cost. Otherwise, the conventional model is preferable.

## 5. Conclusions

This paper deals with an organization's dilemma regarding whether to adopt a cogeneration model in order to produce electricity and steam, or to purchase electricity from the grid and use a conventional boiler to produce steam. In contrast to previous models, we assume that the cogeneration system relies on one heat exchanger, which is used to produce steam both when the cogeneration system is turned on and when it is off. Using backward dynamic programming, we analytically calculate the minimal expected setup and operation costs under the cogeneration model and under the conventional model. In most cases, the operation cost of the cogeneration model is lower than that of the conventional model, while the setup cost is higher. As a result, it is preferable to adopt the cogeneration model for long processes, whereas the conventional model is better for short processes. On the basis of the tariff system used by the Israel Electric Corporation, we numerically find that ROI duration under the cogeneration system may range between 2 and 13 years. Under low cogeneration operation costs or high electricity tariffs, the cogeneration ROI duration is reduced, and the expected cost may be more than 25% lower than the corresponding cost associated with the conventional model. On the other hand, if the operation costs are high or the electricity tariffs are low, ROI duration associated with the cogeneration system increases. Moreover, if the process is short (3 years or fewer), the conventional model is significantly cheaper in most cases, and the cost of the cogeneration system may be more than 20% higher than that of the conventional model. While the total demand for electricity is likely to be close to the total demand for steam (during the whole process), the higher the expected demands, the greater the cost savings due to adopting cogeneration. On the other hand, if the demands for one type of product (electricity or steam) are significantly changed, compared to the other production, then cogeneration becomes less attractive. Especially, if the steam demands grow or the electricity demands are reduced, then the conventional model can save up to 10% of the total cost, even for processes of 5 or 6 years. Moreover, the profitability of each model is affected by the steam demand variations. The lower the variations (namely, more deterministic steam demands), the higher the cost savings due to adopting the cogeneration model.

More than 50% of the time, the Israeli electricity tariff is low. By adding electrical steam generator, it is possible to operate it in order to satisfy site steam demands while the tariffs are low. When tariffs are high, cogeneration should be operated. The electrical steam generator's impact on the results should be considered. Moreover, random electricity tariffs may affect the results and improve the models. We leave these issues for future research.

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